

# Notes on stability of short pulses in the normal regime of the dispersion map

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In this short note, I use the results of my recent paper “Instabilities of dispersion-managed solitons in the normal-dispersion regime” (Phys. Rev. E, September, 2000) to estimate the stability boundary for short pulses in the normal average-dispersion regime of the dispersion map.

We start with a nonlinear Schrödinger equation for the dispersion compensating scheme,

$$i\psi_z - \frac{\beta_2 L}{2\tau^2} \psi_{tt} + \frac{2\pi n_2 L P_0}{\lambda_0 A_{\text{eff}}} |\psi|^2 \psi = 0, \quad (1)$$

where  $L$  is dispersion map’s period,  $n_2$  - nonlinear refractive index,  $\tau$  - characteristic pulse width,  $P_0$  - characteristic power of the input signal,  $\lambda_0$  - operating wavelength,  $A_{\text{eff}}$  - effective fiber area,  $\beta_2 = \beta_2(z)$  is the dispersion coefficient of the fiber. This equation assumes the idealized case of a lossless fiber. There are several physical situations when this approximation is sufficient for modeling such as a fiber with distributed (e.g. Raman) amplification or a fiber with several amplification stages at the dispersion compensation period. In other cases, the results reported here are still expected to hold qualitatively.

We assume  $\beta_2(z) = \beta_2^{\text{av}} + \beta_2^{\text{var}}(z)$ , where  $\beta_2^{\text{av}}$  is the average dispersion and  $\beta_2^{\text{var}}(z)$  is the varying part with zero average and unit period, i.e.  $\beta_2^{\text{var}}(z+1) = \beta_2^{\text{var}}(z)$ . Then, the problem (1) transforms to the dimensionless form,

$$iu_z + \frac{1}{2}D(z)u_{tt} + \epsilon \left( \frac{1}{2}D_0u_{tt} + |u|^2u \right) = 0, \quad (2)$$

where the correspondence between (1) and (2) is

$$D(z) = -\frac{L}{\tau^2}\beta_2^{\text{var}}(z), \quad \epsilon = \frac{2\pi n_2 L P_0}{\lambda_0 A_{\text{eff}}}, \quad D_0 = -\frac{\beta_2^{\text{av}} \lambda_0 A_{\text{eff}}}{2\pi n_2 P_0 \tau^2}, \quad \psi = u.$$

The two-step dispersion map is used for convenient approximation:

$$\beta_2^{\text{var}}(z) = \begin{cases} d_1 & \text{for } 0 \leq z < z_c \\ d_2 & \text{for } z_c \leq z < 1 \end{cases}$$

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where  $z_c = d_2/(d_2 - d_1)$  from the condition that  $\beta_2^{\text{var}}(z)$  has zero average.

Now we consider the principal result of the paper. When  $0 < \epsilon \ll 1$  and  $D_0 < 0$  (i.e.  $\beta_2^{\text{av}} > 0$ ), there are two branches of stationary Gaussian-like solutions of the NLS equation,

$$u(z, t) = C \exp\left(-\frac{\alpha - 2i\beta}{\alpha^2 + 4\beta^2}t^2 + i\phi\right), \quad (3)$$

where  $C$  is the pulse amplitude,  $\phi$  - phase, while parameters  $\alpha$  and  $\beta$  define the pulse minimum width and chirp (see below). The stationary solutions exist in the limit  $\epsilon \rightarrow 0$  for constant values of  $\alpha$  and

$$\beta(z) = -\frac{1}{2}m + \int_0^z D(z')dz', \quad (4)$$

where parameter  $m$  is defined in physical units as

$$m = \left| \frac{d_1 d_2 L}{(d_1 - d_2)\tau^2} \right| > 0. \quad (5)$$

The two branches of the stationary solutions are separated by the threshold  $\alpha = \alpha_{thr}$  so that the stationary pulses are stable for  $\alpha > \alpha_{thr}$  and unstable for  $\alpha < \alpha_{thr}$  (see Figure 2 of the paper). The threshold value  $\alpha_{thr}$  can be found from Eq. (13) of the paper in the form:  $\alpha = m\alpha_*$ , where  $\alpha_*$  is the root of the equation,

$$f(\alpha_*) = \frac{2 + \alpha_*^2}{(1 + \alpha_*^2)^{3/2}} - \frac{3}{4} \ln\left(\frac{1 + (1 + \alpha_*^2)^{1/2}}{\alpha_*}\right) = 0. \quad (6)$$

The numerical value is  $\alpha_* \approx 0.148$ .

We define the effective pulse width  $t_{\text{eff}}$  for the Gaussian pulse according to the expression,

$$|u|^2 = |C|^2 \exp(-2t^2/t_{\text{eff}}^2).$$

At the middle points of the dispersion map, i.e. at  $z = 0.5z_c$  and  $z = 0.5(1 + z_c)$ , the pulse has a minimum width  $t_{\text{eff}}$  which is found from (3) and (4) as

$$t_{\text{eff}}^2 = \alpha. \quad (7)$$

In physical units, the pulse width is  $T_{\text{eff}} = \tau t_{\text{eff}} = \tau\sqrt{\alpha}$ . The threshold (stability) value  $T_{\text{thr}}$  for the pulse width follows from (4) and (5) in the form,

$$T_{\text{thr}}^2 \approx 0.15 \left| \frac{d_1 d_2 L}{(d_1 - d_2)} \right|$$

It is customary to express the parameter  $\beta_2^{\text{var}}(z)$  (i.e. coefficients  $d_1$  and  $d_2$ ) in terms of the associated dispersion parameter  $\mathcal{D}(z)$ ,

$$\beta_2^{\text{var}}(z) = -\frac{\lambda_0^2 \mathcal{D}(z)}{2\pi c},$$

where  $c$  is the speed of light and  $\mathcal{D}$  is measured in ps/(nm x km). Typical values for SMF are  $\mathcal{D} \approx 16 - 17$  ps/(nm x km) and for DCF are  $\mathcal{D} \approx -(85 - 100)$  ps/(nm x km). Typical values for the map's period is  $L = 80$ km and for the wavelength is  $\lambda_0 = 1557$ nm. The speed of light is  $c = 3 \times 10^5$  nm/ps. By using these data, we estimate the threshold pulse width  $T_{\text{thr}}$  as

$$T_{\text{thr}} \approx 14.42\text{ps}.$$

Thus, analytical results predict instability if  $T_{\text{eff}} < 15$  ps. These results are valid for the normal dispersion scheme, i.e. when  $\beta_2^{av} > 0$ , and in the limit of sufficiently small nonlinearities, i.e. when

$$\epsilon = \frac{2\pi n_2 L P_0}{\lambda_0 A_{\text{eff}}} \ll 1. \quad (8)$$

If the stationary pulse (3) is unstable, the instability results in an exponential growth of the perturbations, such that  $\alpha = \alpha_s + \Delta\alpha e^{\epsilon\Gamma z}$ , where  $\Gamma \approx 0.07$  (see Fig. 8 of the paper). The instability becomes visible if  $z > z_{\text{inst}}$ , where

$$z_{\text{inst}} = \frac{1}{\epsilon\Gamma}. \quad (9)$$

Finally, we estimate the characteristic distance where the instability of short pulses could develop. We assume that the power of input signal is  $P_0 = 10$  mW and use the estimates for parameters of a fiber:

$$n_2 = 3 \times 10^{-16} \text{cm}^2/\text{W}, \quad A_{\text{eff}} = \pi r^2,$$

where the radius of the fiber is  $r \approx 1$  mm. These estimates give a very small nonlinearity index:  $\epsilon \approx 3 \times 10^{-5}$  which results in a very long instability distance in  $z_{\text{inst}} = 470,000$  map's periods, i.e. about 40,000,000 km. Of course, this is a linear regime of the dispersion scheme when the instability of short pulses is not visible at the Earth's scale.

However, if the dispersion compensation scheme will be operating in a weakly nonlinear (soliton) regime, i.e. when  $\epsilon \approx 10^{-1}$ , then, the instability of short pulses could develop at  $z_{\text{inst}} \approx 100$  map's periods, i.e. at about 10,000 km. At this scale, local communications inside North America and international communications over the ocean would be greatly affected by the short pulse instabilities.

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