Scalar linear equation

$$ax = b$$

where a and b are numbers and x is an unknown variable.

- If $a \neq 0$, there exists the unique solution $x = b/a$
- If $a = 0$, no solutions exist when $b \neq 0$ and infinitely many solutions $x \in \mathbb{R}$ exists when $b = 0$.

Example

Suppose that a certain diet calls for 9 units of protein and 16 units of carbohydrates for the main meal, and suppose that an individual has two possible foods to choose from to meet these requirements:

A: Each ounce contains 2 units of protein and 4 units of carbohydrates.

B: Each ounce contains 3 units of protein and 5 units of carbohydrates.

How many ounces of each food need to be consumed to meet the requirements?

Linear system is a set of linear algebraic equations of the form:

$$a_1x_1 + a_2x_2 + \cdots + a_nx_n = b \tag{1}$$

where $a_1, a_2, \ldots, a_n$ are called the **coefficients**, the number b is the **constant term**, and $x_1, x_2, \ldots, x_n$ are called **variables**.

A tuple $(s_1, s_2, \ldots, s_n)$ of numbers is called a **solution** of equation (1) if

$$a_1s_1 + a_2s_2 + \cdots + a_ns_n = b.$$

Example

$$2x + 3y = 9$$
$$4x + 6y = 18$$

For arbitrary values of t, the tuple

$$x = t, \; y = 3 - \frac{2}{3}t,$$

is a solution of the system above.

The set of solutions expressed with arbitrary **parameter(s)** is said to be **given in parametric form**.

Since the solution set of *one* linear equation in two variables is a line in 2-space, there are three possibilities for the solution set of *two* linear equations in two variables:

1. The lines intersect in a single point ⇒ **unique solution**.
2. The lines are parallel ⇒ **no solution**.
3. The lines are identical ⇒ **infinitely many solutions**, one for each point on the common line.

Since the solution set of *one* linear equation in three variables is a plane in 3-space, there are three possibilities for the solution set of *two* linear equations in three variables:

1. The planes intersect in a line $\Rightarrow$ **infinitely many solutions**, one for each point on the line.

2. The planes are parallel $\Rightarrow$ **no solution**.

3. The planes are identical $\Rightarrow$ **infinitely many solutions**, one for each point on the common plane.

What about three linear equations in three variables?

1. The planes have no common intersection $\Rightarrow$ **no solution**.
2. The planes intersect in a point $\Rightarrow$ **unique solution**.
3. The planes intersect in a line $\Rightarrow$ **infinitely many solutions**, one for each point on the line.
4. The planes are identical $\Rightarrow$ **infinitely many solutions**, one for each point on the common plane.