General terminology

matrix = a rectangular array of numbers called **entries**

$\mathbf{m \times n}$ **matrix** = matrix with m rows and n columns

row matrix = $1 \times n$ matrix

column matrix = $m \times 1$ matrix

Rows are counted from top to bottom

Columns are counted from left to right

The entry that belongs to row i and column j is called the $\mathbf{(i, j)}$**-entry**.

A general $m \times n$ matrix A:

$$A = \begin{bmatrix} a_{11} & a_{12} & \cdots & a_{1n} \\ a_{21} & a_{22} & \cdots & a_{2n} \\ \vdots & \vdots & & \vdots \\ a_{m1} & a_{m2} & \cdots & a_{mn} \end{bmatrix} = \begin{bmatrix} a_{ij} \end{bmatrix}_{m \times n}.$$

square matrix $= n \times n$ matrix

main diagonal $=$ entries $a_{11}, a_{22}, \ldots, a_{nn}$.

symmetric matrix $= n \times n$ matrix with $a_{ij} = a_{ji}$

Matrix equality

Two matrices A and B are called **equal** if

- They have the same number of rows and columns
- For all i and j, we have $a_{ij} = b_{ij}$.

Example

$$A = \begin{bmatrix} 1+a & 2-b \\ 3+c & 4-d \end{bmatrix} \qquad B = \begin{bmatrix} 0 & 0 \\ 0 & 0 \end{bmatrix}.$$

Matrix additions

If A and B are matrices of the same size, then the sum matrix $A+B$ is the matrix with entries $a_{ij}+b_{ij}$.

Example

$$A = \begin{bmatrix} 1 & 2 \\ 3 & 4 \end{bmatrix} \qquad B = \begin{bmatrix} a & -b \\ c & -d \end{bmatrix}.$$

Theorem Let A, B and C be matrices of the same size.

$$A + B = B + A \quad \text{(commutativity)}$$

$$A + (B + C) = (A + B) + C \quad \text{(associativity)}$$

$$A + O = A \quad \text{(zero matrix)}$$

$$A + (-A) = O \quad \text{(negative matrix)}$$

<u>Example</u>

Solve the matrix equation

$$\begin{bmatrix} 2 & 1 \\ -4 & 6 \end{bmatrix} - X = \begin{bmatrix} 12 & 3 \\ 9 & -7 \end{bmatrix}.$$

Scalar multiplications

For any number k and any matrix $A = [a_{ij}]$, the scalar multiplication matrix kA is the matrix with entries ka_{ij}

Example

$$A = \begin{bmatrix} 2 & 4 \\ -1 & 0 \end{bmatrix} \qquad B = \begin{bmatrix} -2 & 5 \\ 4 & 1 \end{bmatrix}$$

Compute $3A - B$.

Theorem: Let A and B be two matrices of the same size, and let k and p be two numbers. Then:

1. $k(A+B) = kA + kB$.
2. $(k+p)A = kA + pA$.
3. $(kp)A = k(pA)$.

<u>Example</u>

Simplify $2(A + 3B - 4C)$

<u>Example</u>

Find all solutions of the matrix equation $kA = O$.

Transpose

If $A = [a_{ij}]$ is a $m \times n$ matrix, then its **transpose** A^T is the $n \times m$ matrix with the entries

$$A^T := \left[a_{ji} \right].$$

Symmetric $n \times n$ matrix A is the matrix with $A^T = A$.

Example

$$A = \begin{bmatrix} 2 & 15 & -1 \\ 3 & 4 & 5 \end{bmatrix} \qquad A = \begin{bmatrix} 3 & 2 & 1 \\ 2 & 5 & 4 \\ 1 & 4 & 0 \end{bmatrix}$$

Theorem: Let A and B be matrices of the same size, and let k be a number. Then:

1. $(A^T)^T = A$.
2. $(kA)^T = kA^T$.
3. $(A + B)^T = A^T + B^T$.

<u>Example</u>

If A and B are symmetric matrices of the same size, prove that $kA + pB$ is a symmetric matrix.