Projections: Let $\mathbf{d}$ be a nonzero vector, and let $\mathbf{u}$ be any vector. The **projection** of $\mathbf{u}$ on $\mathbf{d}$, denoted by $\mathbf{proj_d}(\mathbf{u})$, is the vector, which is parallel to $\mathbf{d}$ with the length given by $\|\mathbf{u}\| \cos\theta$, where θ is the angle between $\mathbf{u}$ and $\mathbf{d}$.

Properties

1. The vector $\mathbf{proj_d}(\mathbf{u})$ is parallel to $\mathbf{d}$.
2. The vector $\mathbf{v} := \mathbf{u} - \mathbf{proj_d}(\mathbf{d})$ is orthogonal to $\mathbf{d}$.
3. The vector $\mathbf{u}$ is decomposed as $\mathbf{u} = \mathbf{proj_d}(\mathbf{u}) + \mathbf{v}$.

Example:

$$\mathbf{d} = \begin{pmatrix} 4 \\ 0 \\ -1 \end{pmatrix}, \qquad \mathbf{u} = \begin{pmatrix} 2 \\ -1 \\ 1 \end{pmatrix}$$

Theorem: Given $\mathbf{u}$ and $\mathbf{d} \neq \mathbf{0}$. Then

$$\mathbf{proj}_{\mathbf{d}}(\mathbf{u}) = \frac{\mathbf{u} \cdot \mathbf{d}}{\|\mathbf{d}\|^2}\mathbf{d}.$$

Example: Calculate the distance from a point $P(3, 2, -1)$ to the line

$$\begin{aligned} x &= 2 + 3t, \\ y &= 1 - t \\ z &= 3 - 2t \end{aligned}$$