n-dimensional vector space $\mathbf{R}^n$

$\mathbf{R}^n$ is a set of all n-**vectors** ($n \times 1$-column matrices) $\mathbf{v}$ with coordinates (components) $(v_1, v_2, \ldots, v_n)$, where all numbers are real.

Example:

$$\mathbf{R}^1$$

$$\mathbf{R}^2$$

$$\mathbf{R}^3$$

Example: Generalize a straight line in $\mathbf{R}^n$ for $n \geq 2$.

Subspace

A set U of vectors of $\mathbf{R}^n$ is called a **subspace** of $\mathbf{R}^n$ if it has the following three properties:

S1. The zero vector $\mathbf{0}$ belongs to U.

S2. If $\mathbf{u}$ and $\mathbf{v}$ are in U, then $\mathbf{u}+\mathbf{v}$ belongs to U.

S3. If $\mathbf{u}$ is in U and k is a real number, then $k\mathbf{u}$ belongs to U.

Trivial subspaces:

$$U = \mathbf{R}^n$$

$$U = \{\mathbf{0}\}$$

Proper subspaces:

- $U = \{$all lines in $\mathbf{R}^n$ through the origin$\}$
- $U = \{$all planes in $\mathbf{R}^3$ through the origin$\}$
- $U = \{$all solutions of $AX = O\} = \text{null}(A)$, where A is a $m \times n$ matrix and X is a $n \times 1$ matrix.

Example

$$\begin{aligned} x_1 + x_3 + x_4 &= 0 \\ 2x_1 + x_2 - x_4 &= 0 \end{aligned}$$