0. Linear systems of mathematical physics

- Algebraic systems of linear equations

$$\begin{aligned} a_{11}x_1 + a_{12}x_2 &= b_1 \\ a_{21}x_1 + a_{22}x_2 &= b_2 \end{aligned}$$

- Linear (ordinary) differential equations

$$y'' + p(x)y' + q(x)y = r(x), \qquad a \leq x \leq b$$

- Linear (partial) differential equations

$$u_{tt} = c^2(u_{xx} + u_{yy})$$

- Linear integral transforms

$$\begin{aligned} f(x) &= \frac{1}{2\pi}\int_{-\infty}^{\infty} \hat{f}(k)e^{-ikx}dk \\ \hat{f}(k) &= \int_{-\infty}^{\infty} f(x)e^{ikx}dx \end{aligned}$$

Linear operators $\mathcal{A}(v)$ on a vector space $v \in \mathbf{V}$ satisfy two main properties:

1. $\forall v_1, v_2 \in \mathbf{V}: \qquad \mathcal{A}(v_1 + v_2) = \mathcal{A}(v_1) + \mathcal{A}(v_2)$
2. $\forall v \in \mathbf{V},\ \lambda \in \mathbb{R}: \qquad \mathcal{A}(\lambda v) = \lambda \mathcal{A}(v)$

Linear Superposition Principle: If v_1 and v_2 are two particular solutions of the linear homogeneous system $\mathcal{A}(v) = 0$, then $v = c_1 v_1 + c_2 v_2$ is also a solution of the same system $\mathcal{A}(v) = 0$ for any $c_1, c_2 \in \mathbb{R}$.