

# MATH 3C03: Home Assignment # 4

**Due to:** November 13, 2014

**Problem 1:** Express the function  $f(x) = \cos(\pi x)$  given for  $x \in (0, 1)$  as Fourier sine and cosine series. Plot the Fourier sine and cosine series for  $x \in (-1, 1)$ .

**Problem 2:** Use the Fourier series to solve the boundary-value problem

$$\frac{d^2 y}{dx^2} + y(x) = x, \quad x \in (0, 1),$$

subject to the boundary conditions  $y'(0) = y'(1) = 0$ .

**Problem 3:** Solve the heat equation

$$\frac{\partial u}{\partial t} = \frac{\partial^2 u}{\partial x^2}, \quad x \in (0, 1), \quad t > 0,$$

subject to the boundary conditions  $u(0, t) = u(1, t) = 0$  and the initial condition  $u(x, 0) = 1$  for all  $x \in (0, 1)$ .

**Problem 4:** Consider the damped wave equation

$$\frac{\partial^2 u}{\partial t^2} + \gamma \frac{\partial u}{\partial t} = \frac{\partial^2 u}{\partial x^2}, \quad x \in (0, 1), \quad t > 0,$$

where  $\gamma > 0$  is the damping coefficient. Solve this equation subject to the boundary conditions  $u(0, t) = u(1, t) = 0$  and the initial condition  $u(x, 0) = 2 \sin(\pi x)$  and  $u_t(x, 0) = 0$ . Assume that  $\gamma < 2\pi$  and use Fourier harmonics (normal modes).

**Problem 5:** Consider the Laplace equation in the square:

$$\frac{\partial^2 u}{\partial x^2} + \frac{\partial^2 u}{\partial y^2} = 0, \quad x \in (0, 1), \quad y \in (0, 1),$$

subject to the boundary conditions

$$u(0, y) = 0, \quad u(1, y) = 1, \quad u(x, 0) = 0, \quad u(x, 1) = -1.$$

Find the unique solution by using the principle of linear superposition.

**Problem 6:** Consider the Laplace equation in polar coordinates in the unit disk

$$\frac{\partial^2 u}{\partial r^2} + \frac{1}{r} \frac{\partial u}{\partial r} + \frac{1}{r^2} \frac{\partial^2 u}{\partial \theta^2} = 0, \quad r \in (0, 1), \quad \theta \in (0, 2\pi),$$

subject to the boundary condition  $u(1, \theta) = f(\theta)$  on the unit circle. Use the Fourier series solutions of the boundary-value problem to prove the mean value property

$$u|_{r=0} = \frac{1}{2\pi} \int_0^{2\pi} u(1, \theta) d\theta = \frac{1}{2\pi} \int_0^{2\pi} f(\theta) d\theta.$$

Note that  $u|_{r=0}$  is independent on  $\theta$ , which is not defined at  $r = 0$ .