# MATH 3C03: Home Assignment # 5

**Due to:** November 27, 2014

**Problem 1**: Solve the Laplace equation $\Delta u = 0$ inside a sphere of radius $R$ under the boundary condition $u|_{r=R} = 5\cos^2(\theta)$, where $r$ is the radial variable and $\theta$ is the latitudinal angle in spherical coordinates.

**Problem 2**: Solve the heat equation $u_t = u_{xx} + u_{yy}$ for $(x, y)$ in the unit square subject to the Neumann boundary conditions and the initial condition $u|_{t=0} = \cos(\pi x)\cos^3(\pi y)$.

**Problem 3**: Find the inverse Fourier transform for the function

$$\hat{f}(k) = \frac{2}{8 + 4k + k^2}.$$

**Problem 4**: Determine the double Fourier transform of the two-dimensional Gaussian function

$$f(x, y) = \frac{1}{4\pi t} e^{-\frac{x^2+y^2}{4t}}, \quad t > 0.$$

**Problem 5**: Solve the heat equation $u_t = u_{xx}$ for $x$ on the infinite line subject to the initial condition $u|_{t=0} = e^{-x^2}$.

**Problem 6**: Solve the heat equation with a point source

$$\begin{cases} \frac{\partial u}{\partial t} = \frac{\partial^2 u}{\partial x^2} + \alpha\delta(x), \quad x \in \mathbb{R}, \quad t > 0, \\ u|_{t=0} = 0, \end{cases}$$

where $\alpha \in \mathbb{R}$ is parameter and $\delta(x)$ is the Dirac delta distribution.