MATH 3J04: Home Assignment # 4

Due to: November 7, 2000

Problem 11.3 #4: Consider the string of length $L = \pi$. The ends of the string are fixed at $x = 0$ and $x = L$. The initial velocity is zero and the initial deflection is

$$f(x) = 0.1x(\pi - x)$$

Find the vertical deflection $u(x,t)$ at later times by solving the wave equation:

$$u_{tt} = u_{xx}$$

Problem 11.4 #19: Consider an elastic bar with initial displacement $u(x,0) = f(x)$ and zero initial velocity. The bar is fastened at one end, $x = 0$, and is free at the other end, $x = L$. The boundary conditions for displacement $u(x,t)$ are

$$u(0,t) = 0 \quad u_x(L,t) = 0$$

Find the displacement $u(x,t)$ at later times by solving the wave equation:

$$u_{tt} = u_{xx}$$

Problem 11.5 #4: Consider the bar of length $L = 10$, whose ends are kept at temperature zero and whose initial temperature is

$$f(x) = k\sin(0.2\pi x)$$

Find the temperature $u(x,t)$ at later times by solving the heat equation:

$$u_t = u_{xx}$$

Problem 11.5 #18(a):Consider the square plate

$$S = \{0 \leq x \leq a, 0 \leq y \leq a\}$$

with $a = 2$. Find steady-state solutions of the Laplace equation,

$$u_{xx} + u_{yy} = 0$$

for the following boundary conditions:

$$u(x,a) = \sin(\pi x), \quad u(x,0) = u(0,y) = u(a,y) = 0$$

Problem 11.6 #2: Consider the heat equation $u_t = u_{xx}$ for the initial condition $u(x,0) = f(x)$, where

$$f(x) = \frac{1}{1+x^2}$$

Find the solution $u(x,t)$ of the problem by using the Fourier Cosine transform.

Problem 11.8 #12: Find the deflection $u(x,y,t)$ of the square membrane with $a = b = 1$ and $c^2 = 1$ if the initial velocity is zero and the initial deflection is

$$U(x,y,0) = f(x,y) = k\sin(\pi x)\sin(\pi y),$$

where k is constant.