MATH 3J04: Home Assignment # 5

Due to: November 21, 2000

Problem 19.4 #4: Consider the Laplace equation $u_{xx} + u_{yy} = 0$ on the grid $x = 0, 1, 2, 3$ and $y = 0, 1, 2, 3$ (Fig. 424, p.970). Given the boundary values $u = 0$ on the left edge, $u = x^3$ on the lower edge, $u = 27 - 9y^2$ on the right edge, and $u = x^3 - 27x$ on the upper edge, find a solution to the Laplace equation by a finite difference method. Solve the resulting linear system at the four interior points either by Gauss elimination or by Gauss-Seidel method.

Problem 19.6 #6: Consider the heat equation $u_t = u_{xx}$ on the grid $x = 0, 0.2, 0.4, 0.6, 0.8, 1.0$ and $t = 0, 0.01, 0.02, 0.03, 0.04, 0.05$. Given the zero boundary values at $x = 0$ and $x = 1$ and the initial values: $u = x$ for $0 \leq x \leq 0.5$ and $u = 1 - x$ for $0.5 \leq x \leq 1$, find a solution of the heat equation by the explicit method. Use the symmetry of initial condition $u(x,0) = u(1-x,0)$ to reduce the number of computations by half.

Problem 19.6 #8: Consider the heat equation $u_t = u_{xx}$ on the grid $x = 0, 0.2, 0.4, 0.6, 0.8, 1.0$ and $t = 0, 0.01, 0.02, 0.03, 0.04, 0.05$. Given the boundary values $u_x = 0$ at $x = 0$ (the left end is insulated) and $u = sin(50\pi t/3)$ at $x = 1$ (the right end is kept at a given temperature) and the zero initial value at $t = 0$, find a solution of the heat equation by the explicit method.

Problem 19.7 #2: Consider the wave equation $u_{tt} = u_{xx}$ on the grid $x = 0, 0.2, 0.4, 0.6, 0.8, 1.0$ and $t = 0, 0.2, 0.4, 0.6, 0.8, 1$. Given the zero boundary values at $x = 0$ and $x = 1$ and the initial values: $u = x^2(1 - x)$, $u_t = 0$ for $t = 0$, find a solution of the wave equation by the explicit method.

Problem 22.3 #6: What is the probability of obtaining at least one Six in rolling three fair dice?

Problem 22.5 #14: Find the probability function of $X = n$, where $n = 1, 2, ...$ is *number of times a fair die is rolled until the first Six appears.* Show that the probability function $f(x) = p_n$ satisfies the normalization condition:

$$\sum_{n=1}^{\infty} p_n = 1.$$