

Math 3J04: Term Test # 1
October 11, 2000

FAMILY NAME: _____

GIVEN NAME(S): _____

STUDENT NUMBER: _____

SIGNATURE: _____

Instruction: No aids allowed. The duration of this test is 50 minutes.

This test has 3 questions, where the marks are specified next to each question. Total marks = 15. Write solutions in the spaces provided, using the backs of the pages if necessary. Show your work.

1. Consider the system of differential equations:

$$\frac{dy_1}{dt} = 2y_2 + 2y_2^2$$

$$\frac{dy_2}{dt} = -8y_1$$

[2] (a) Find all critical points of the system. Linearize the system near the critical points.

[2] (b) Find general solutions for the linearized systems in (a).

[1] (c) Characterize the type of the critical points in (a).

2. Consider the non-homogeneous system of differential equations:

$$\frac{dy_1}{dt} = -4y_1 + y_2 + 4$$

$$\frac{dy_2}{dt} = y_1 - 4y_2 - 1$$

[2] (a) Find a particular (constant) solution of the system.

[2] (b) Solve the system: find a general solution of the system.

[1] (c) Select from the general solution in (b) a particular solution that satisfies the initial values: $y_1(0) = 0$, $y_2(0) = 0$.

[1] (bonus) (d) Set a Gauss-Seidel iterative scheme for finding a particular (constant) solution in (a). Do not actually run the iterations.

3. Consider the linear eigenvalue problem $A\mathbf{x} = \lambda\mathbf{x}$, where

$$A = \begin{pmatrix} 0 & 4 & 0 \\ -4 & 0 & 3 \\ 0 & -3 & 0 \end{pmatrix}, \quad \mathbf{x} = \begin{pmatrix} x_1 \\ x_2 \\ x_3 \end{pmatrix}.$$

[1] (a) How many eigenvalues does the matrix A have? Are the eigenvalues real, pure imaginary or complex-valued? Are the eigenvectors real, pure imaginary or complex-valued?

[2] (b) Find the spectrum and eigenvectors of the linear eigenvalue problem.

[2] (c) Find the vector $\mathbf{y} = (y_1, y_2, y_3)$, for which the matrix A is diagonal. Write the diagonalization formula for A .