Math 3J04: Term Test # 2
November 8, 2000

FAMILY NAME: ______________________________

GIVEN NAME(S): ____________________________

STUDENT NUMBER: __________________________

SIGNATURE: _________________________________

Instruction: No aids allowed. The duration of this test is 50 minutes.

This test has 3 questions, where the marks are specified next to each question. Total marks = 15. Write solutions in the spaces provided, using the backs of the pages if necessary. Show your work.

1. Consider a bar of length $L = 10$ whose endpoints are kept at zero temperature. Mathematical modeling of the temperature in the bar is based on the heat equation:

$$u_t = u_{xx}, \quad 0 \le x \le 10, \quad t \ge 0$$

with the boundary conditions

$$u(0,t) = u(10,t) = 0$$

and the initial condition

$$u(x,0) = f(x).$$

[2] (a) Find a Fourier sine-series which solves the problem above.

[2] (b) Match the Fourier amplitudes of the sine-series with the Fourier coefficients of the initial temperature:

$$f(x) = 5 - |5 - x|, \quad 0 \le x \le 10.$$

[1] (c) Write down the first three terms of the Fourier solution of the problem. Explain what is happening with the temperature distribution when time becomes larger, in the limit $t \to \infty$.

2. Consider the following function

$$f(x) = \begin{cases} e^{-2x} & x > 0 \\ 0 & x < 0 \end{cases}$$

[1] (a) Can the function be expanded into Fourier Cosine Transform? Fourier Sine Transform? General Fourier transform? Give a reason.

[2] (b) Find the appropriate Fourier transform $F(\omega)$ of the function $f(x)$.

[2] (c) Explain what value of the function $f(x)$ is reproduced by the Fourier transform of $F(\omega)$ at $x = 0$. Use the following integral as given:

$$\frac{1}{2\pi i}\int_{-\infty}^{\infty} \frac{e^{i\omega x} d\omega}{\omega - 2i} = \begin{cases} e^{-2x} & x > 0 \\ 1/2 & x = 0 \\ 0 & x < 0 \end{cases}$$

3. Consider the following initial-value problem:

$$\frac{d^2y}{dt^2} = t^2 y, \quad y(0) = 1, \quad y'(0) = 0.$$

[2] (a) Write down the Euler method to solve the problem.

[2] (b) Write down the improved Euler method to solve the problem.

[1 + 1(bonus)] (c) Apply the first two steps of each of the methods with the time step $\Delta t = 1$.