

MATH 4X03: Home Assignment # 5

Due to: November 21, 2000

Problem 1: Find the location of the branch points and discuss possible branch cuts for the following functions:

- (a) $f(z) = \frac{1}{(z-1)^{1/2}}$
- (b) $f(z) = (z+1-2i)^{1/4}$
- (c) $f(z) = 2 \log z^2$

Problem 2: Find the location of the branch points and discuss possible branch cuts for the following functions:

- (a) $f(z) = ((z-1)(z-2))^{1/3}$
- (b) $f(z) = \log((z-1)(z-2))$

Problem 3: The function $f(z)$ is defined in the domain $|z| < 1$ by the Taylor series

$$f(z) = \sum_{n=1}^{\infty} \frac{z^n}{n}$$

Find the analytic continuation of the function to the domain $|z| > 1$.

Problem 4: Discuss all (isolated and non-isolated) singularities of the following functions:

- (a) $f(z) = \frac{\log(z+1)}{(z-1)}$
- (b) $f(z) = \frac{z^{1/3} - 1}{z-1}$

Problem 5: Use the keyhole contour to find the following integrals on the principal branch of the function z^k :

- (a) $\int_0^{\infty} \frac{x^{k-1}}{(x+a)} dx$
- (b) $\int_0^{\infty} \frac{x^{1/2}}{(1+x^2)} dx$