MATH 2Z03: Test 1 Group A - Version 1
Instructor: Livia Corsi, Eric Harper, David Lozinski
Date: Thursday, October 16
Duration: 90 minutes.

Name: _____ **ID #:** _____

Instructions:
This test paper contains 13 multiple choice and short answer questions printed on both sides of the page. The questions are on pages 2 through 13. Pages 14 to 16 are available for rough work. YOU ARE RESPONSIBLE FOR ENSURING THAT YOUR COPY OF THE PAPER IS COMPLETE. BRING ANY DISCREPANCIES TO THE ATTENTION OF THE INVIGILATOR.

Select the one correct answer to each question and ENTER THAT ANSWER INTO THE SCAN CARD PROVIDED USING AN HB PENCIL. Room for rough work has been provided in this question booklet. You are required to submit this booklet along with your answer sheet, However, only short answer questions will be graded from the booklet. Point values are marked on each question. The test is graded out of 45.
SHORT ANSWER QUESTIONS MUST BE COMPLETED IN PERMANENT INK.
Computer Card Instructions;

IT IS YOUR RESPONSIBILITY TO ENSURE THAT THE ANSWER SHEET IS PROPERLY COMPLETED. YOUR TEST RESULTS DEPEND UPON PROPER ATTENTION TO THESE INSTRUCTIONS.

The scanner that will read the answer sheets senses areas by their non-reflection of light. A heavy mark must be made, completely filling the circular bubble, with an HB pencil. Marks made with a pan or felt-tip marker will **NOT** be sensed. Erasures must be thorough or the scanner may still sense a mark. Do **NOT** use correction fluid.

- Print your name, student number, course name, and the date in the space provided at the top of Side 1 (red side) of the form. Then the sheet **MUST** be signed in the space marked SIGNATURE.
- Mark your student number in the space provided on the sheet on Side 1 **and fill the corresponding bubbles underneath**.
- Mark only **ONE** choice (A, B, C, D, E) for each question.
- Begin answering questions using the first set of bubbles, marked "1".

1. (3 marks) Which of the following could be used to describe the differential equation

$$\cos(x)y''' + e^{2x}y = xy \ ?$$

(a) linear, first-order, homogeneous

(b) autonomous, third-order, nonhomogeneous

(c) non-linear, third-order, nonhomogeneous

(d) linear, third-order, homogeneous

(e) autonomous, first-order, nonhomogeneous

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2. (3 marks) Which of the following could be a solution to the IVP

$$\frac{dy}{dx} + \sin(x)y = 0, \ y(0) = \pi \ ?$$

(a) $y = e^{\int_0^x \sin(s)\,ds}$

(b) $y = e^{\int_0^x \cos(s)\,ds}$

(c) $y = ce^{\int_0^\pi \sin(x)\,dx}$

(d) $y = \pi e^{\cos(x)-1}$

(e) $y = \pi e^{\cos(x)}$

3. (3 marks) Which of the following most accurately describes the critical points of the autonomous differential equation

$$\frac{dy}{dx} = y(y+2)(y-2)^2 \ ?$$

(a) -2 is stable, 0 is unstable, 2 is semi-stable

(b) -2 is unstable, 0 is unstable, 2 is semi-stable

(c) -2 is stable, 0 is stable, 2 is stable

(d) -2 is unstable, 0 is unstable, 2 is stable

(e) -2 is stable, 0 is semi-stable, 2 is stable

4. (3 marks) What technique would be best used to solve the following differential equation

$$yy' + e^{2x}y^2 = e^{2x} \ ?$$

   (a) Integrating Factor

   (b) Separation of Variables

   (c) Variation of Parameters

   (d) Integration by Parts

   (e) Undetermined Coefficients

5. (3 marks) Euler's method can be used to obtain an approximate numerical solution to an initial value problem of the form

$$y' = f(x, y),\ y(x_0) = y_0.$$

Given starting values x_0 and y_0, and step size h, the approximations $y_{n+1} \approx y(x_{n+1})$ are obtained from the recursive formula,

$$y_{n+1} = y_n + hf(x_n, y_n),\ n = 0, 1, 2, \ldots$$

where $x_n = x_0 + nh$, or equivalently $x_{n+1} = x_n + h$.

Let $y(x)$ be the solution to the following IVP

$$y' = 4x + y,\ y(0) = 18$$

Consider the following MATLAB code to approximate the value $y(2)$:

```
h = 0.4;
X = 2;
N = round(X/h);
== missing code ==
x(n+1) = x(n) + h;
y(n+1) = y(n) + h*(4*x(n) + y(n));
end
```

Which of the following is the correct "== missing code ==" from the above?

(a)
```
x(0) = 1; y(0) = 18;
for n = 1:N
```

(b)
```
x(1) = 0; y(1) = 18;
for n = 1:N
```

(c)
```
x(1) = 0; y(0) = 18;
while n = 1:N
```

(d)
```
x(0) = 0; y(1) = 18;
for n = 1:N
```

(e)
```
x(0) = 0; y(0) = 18;
while n = 1:N
```

6. (3 marks) Given the linear ODE

$$y'' - 4y' + 4y = 0$$

which is its general solution?

(a) $c_1 e^x + c_2 e^{-4x}$

(b) ce^{2x}

(c) $c_1 e^{-x} + c_2 e^{-4x}$

(d) $c_1 \cos 4x + c_2 \sin 4x$

(e) $(c_1 + c_2 x)e^{2x}$

7. (3 marks) Consider the following Boundary Value Problem

$$\begin{cases} y'' = ky \\ y(0) = y(2\pi) = 0 \end{cases}$$

For what value of k does the BVP admit a non-trivial solution?

(a) $k = 1$

(b) $k = -9$

(c) $k = 0$

(d) $k = \pi$

(e) $k = 4$

8. (3 marks) Consider the following ODE

$$\sin(x)y' + 3x^2y = x$$

For which initial condition does the solution not exist?

(a) $y(3) = 0$
(b) $y(\pi) = 0$
(c) $y(1) = 3$
(d) $y(-3) = 0$
(e) $y(-1) = -3$

9. (3 marks) The temperature of a cupcake at time t is given by $T(t)$, and the temperature follows Newton's Law of Cooling. The surrounding air is at a constant 20 degrees, while the cupcake begins at a temperature of 30 degrees. If, at time $t = 2$, the cupcake has a temperature of 26.4 degrees, what temperature is the cupcake at time $t = 3$?

   (a) 24.82

   (b) 23.2

   (c) 24.6

   (d) 23.15

   (e) 25.12

10. (3 marks) A population size given by the function $P(t)$ satisfies the ODE

$$\frac{dP}{dt} = f(P)P$$

Which function $f(P)$ would produce a model for which a small intial population of $P(0) = 5$ would grow to approach a finite maximum value?
DO NOT SOLVE FOR THE SOLUTION TO THE ODE.

(a) $f(x) = \cos(x/1000)$

(b) $f(x) = e^{(x/1000)}$

(c) $f(x) = -(1000 - x)$

(d) $f(x) = x/1000$

(e) $f(x) = -\tan(x/1000)$

11. (3 marks) A linear inhomogeneous ODE has a particular solution of

$$2e^{2t} + 4e^{4t}$$

The associated homogeneous ODE has (complementary) solutions that include

$$2e^{2t} - 3e^{3t}$$

and

$$2e^{2t} + 3e^{3t}$$

Which of the following is another solution to the original nonhomogeneous ODE?

(a) $8e^{2t} - 12e^{3t}$

(b) $3e^{3t} - 4e^{4t}$

(c) $2e^{2t} - 4e^{4t}$

(d) $6e^{3t} + 4e^{4t}$

(e) $3e^{3t} - 2e^{2t}$

FOR THE FOLLOWING; PROVIDE THE SOLUTIONS ON THIS QUESTION BOOKLET. YOU MUST SHOW ALL YOUR WORK FOR FULL CREDIT.

12. (6 marks.) Find the general solution to

$$y'' + 2y' + 17y = 9$$

13. (6 marks.) Use the method of undetermined coefficients to find $y(x)$; the solution to the following Initial Value Problem

$$\begin{cases} y' = -y + \sin x \\ y(0) = 1 \end{cases}$$

END OF TEST QUESTIONS

Extra page for rough work. DO NOT DETACH!

Extra page for rough work. DO NOT DETACH!

Extra page for rough work. DO NOT DETACH!

END OF TEST PAPER